Finite element modelling and simulation of thermo-elastical damping of MEMS vibrations

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ABSTRACT

The contribution is directed to providing accurate simulation and approximation of the Q-factor determined by thermal-elastic damping in complex micro-electromechanical (MEM) resonators. The base model created is presented as a system of partial differential equations, which describe the elastic and thermal phenomena in the MEM structure. The FEM calculations were performed by using COMSOL Multiphysics software. The model was verified by comparing numerically and analytically obtained damped modal properties of a MEM cantilever resonator. The comparison of calculated and experimentally obtained resonant frequencies and Q-factor values indicated a good agreement of tendencies of change of the quantities against temperature. Investigation of longitudinal and bending vibration modes in 3D of a beam resonators was accomplished by taking into account the layered structure of the resonator and the influence of the geometry of the clamping zone. Modal properties of rectangle- and ring-shaped bulk-mode MEM resonators were examined, too.
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Keywords: MEMS vibrations, thermo-elastical damping, modeling

1. INTRODUCTION

The scope of MEMS is expanding rapidly. Besides traditional MEMS, many new frontiers of practice were opened up in recent years[1]. The most advanced solutions in terms of MEMS sensors, actuators, mechanical filters or microfluidic systems can be identified in inertial navigation systems, computer devices, industrial process control, electronics instrumentation, telecommunications as well as in biological and medical applications.

Advances in electronic-mechanical integration of sensors and utilisation of powerful sensor fusion and navigation algorithms have made possible the embodiment of a wide variety of solutions (with an integrated navigation capability) for tracking and navigation of small platforms for lengthy periods[2]. However, size, weight, cost and power issues remain limiting factors for near-term incorporation of such devices in applications in which space and power are limited. In particular, MEMS inertial measurement units (IMU) drift rates are still too high for use without GPS or another source of position updates.

To improve system performance the ways are being explored for reducing the physical dimensions, weight, power dissipation and noise as well as for increasing sensitivity and accuracy of both mechanical microstructures and electronic components by using innovative combinations of low-power circuits and micromechanical devices[3,4].

In many applications, the benefits of using MEM resonator structures relate directly to the small size, the (relatively) high frequency and the spectral purity. The latter quantity is defined by high values of the mechanical quality factor, the Q-factor. The damping rate of the vibrations can be evaluated as Q^{-1}, which is the rate at which energy of resonant vibrations is being lost to various environments coupled to the resonator.

As an illustration of this viewpoint, an example can be given of the JPL MEMS gyroscope effort that has been aimed at achieving comparable performance to optical gyroscopes while retaining all the advantages of MEMS devices[5].

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High-quality factor quartz disc resonators have been fabricated with Q's ~50x greater (i.e., $Q \sim 8 \times 10^5$) than silicon resonators of the same geometry, enabling increased device sensitivity of any resonant vibratory MEMS device.

The vibrations of actual resonator structures are affected by several different energy loss mechanisms, which predetermine the overall value of the Q-factor. Unfortunately, the physical mechanisms of dissipation affecting the Q-factor values of micro-resonators are still not completely understood. It appears that in-depth theoretical models and analysis are needed to reveal underlying different energy loss mechanisms and to determine the dominating energy dissipation processes. Several studies[6-15], have examined the different dissipation mechanisms, as well as, the dependence of the Q-factor on various parameters in both flexural and longitudinal vibration modes of MEMS structures.

In particular, the Q-factor determined by thermo-elastic damping (TED) of MEM resonator structures, designated as Q_{TED}, is a very important dynamic characteristic since it provides the upper limit of the overall Q-factor that can be attained in a structure of a given geometry and materials under the assumption that no internal friction losses and other sources of damping are present.

In[9] the TED of the single-crystal 3C-SiC UHF nano-mechanical rod resonators at longitudinal vibration mode have been investigated. The TED was studied for thermally insulated boundary conditions as well as for fixed temperature boundary conditions. The Q_{TED} was determined for these conditions. The results from this research were used for the development and verification of FEM computational models[6-8].

In[10] an extensive study of vibration energy loss mechanisms, which limit the highest achievable Q-factor values, is presented. The damping mechanisms are highly influenced by the design of the resonator structures. The authors have included energy loss mechanisms due to support clamping, air damping, heating, TED dissipation, anharmonic mode coupling, surface roughness, extrinsic noise, dislocations and dissipation due to two-level systems. Dependence of dissipation has been analyzed for dependence on finite size of the structure, temperature, surface-aggregated defects, magnetic field, and hysteresis amongst other factors. The authors present data on dissipation measurements for micron-sized single crystal GaAs and Si resonators.

In[11] intrinsic and extrinsic energy loss mechanisms have been discussed and dissipation in polycrystalline diamond (poly-C) resonators has been explored by using electrostatic and piezoelectric actuation methods.

An effort to delineate the microscopic mechanisms predominantly responsible for dissipation in micromechanical resonators has been presented in[12]. Possible mechanisms contributing to dissipation in a double-clamped beam ultra-high frequency (UHF) nano-resonators have been analyzed in[13]. Estimation of the dissipation contributed by the evaporated metallic layers (Al and Ti) with internal friction has been presented in[14].

In the aforementioned studies the numerous approaches to the damping theory and experiments have been systematically developed and reported over several decades, but advanced modeling and simulation seem lacking whereas these are still of relevance in order to provide „advisory service" at early stages of MEMS design. The numerical simulation is important for two reasons. Firstly, it can be used to verify the analytical results, which often are obtained on the base of highly simplified models. And secondly, for complex geometries or stacks of multiple materials there are simply no analytical approximations. Therefore, for these cases, numerical simulations are still necessary.

This work presents the study of TED as a dissipation mechanism in modal damping of vibrations of MEMS resonators by taking into account the layered structure of the resonator and the influence of the geometry of the clamping zone. The dissipation was studied by performing numerical simulations in a finite element package.

The numerical model was verified by relating the simulated data to measurements of SOI-based resonators. Modal properties of square- and ring-shaped bulk-mode MEM resonators were examined, too. The verification demonstrated a reasonable agreement between computationally determined features and physical measurements.

2. MEMS MODELLING

Throughout the course of the project LTU-AVT-05/1 (2006-2008) and LTU-07/1 (2009-2011) a method and model were developed to MEMS dynamic performance assessment, where the problem of the thermo-elastic damping of MEMS resonators was addressed. The thermal-mechanical coupling has been ensured by entering both the body heat source, which described the generation of the heat in the volume at given strain rates as $-\frac{T_0 \alpha E}{1-2\sigma}\left(\frac{\partial \dot{u}}{\partial x}+\frac{\partial \dot{v}}{\partial y}+\frac{\partial \dot{w}}{\partial z}\right)$, and the thermal expansion strain $\alpha\left(T-T_0\right)$, which was taken into account in the mechanical equations of the model. Here

$u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$ are displacements of the MEM resonator structure at location (x,y,z), E - stiffness (or Young's) modulus, σ - Poisson's ratio, α - thermal expansion coefficient, T_0 - ambient temperature.

The model enabled us to take into consideration the TED effect and the influence of the geometry of the clamping zone. The mechanically free surfaces of the resonator were assumed to be thermally isolated. The assumption was based on the fact that the heat exchange rate through the surface is too slow to be comparable with the quick internal processes of acquiring and loosing heat caused by high rates of elastic strain. The fixed temperature boundary conditions as $T = T_0$ were imposed on cut-boundaries, which represented the contact of the resonator with the anchors and with the substrate.

The finite element model is presented as a system of structural dynamic equations. The second order differential equation system has been transformed to the first order one by performing substitution $\{\mathbf{V}\} = \{\dot{\mathbf{U}}\}$, where $\{\mathbf{U}\}$ is the nodal displacement vector. The eigenvalue problem, from which structural modes of the MEM resonator were calculated, reads as

$$\det\left(\begin{bmatrix} [\mathbf{C}] & [\mathbf{K}] & -[\mathbf{H}] \\ -[\mathbf{I}] & 0 & 0 \\ T_0[\mathbf{H}]^T & 0 & [\mathbf{K}_T] \end{bmatrix} + \lambda \begin{bmatrix} [\mathbf{M}] & 0 & 0 \\ 0 & [\mathbf{I}] & 0 \\ 0 & 0 & T_0\left[\tilde{\mathbf{C}}_T\right] + [\mathbf{C}_T] \end{bmatrix}\right) = 0, \quad (1)$$

where $[\mathbf{M}]$, $[\mathbf{K}]$ - mass and stiffness matrices; $[\mathbf{C}] = a_1[\mathbf{M}] + a_2[\mathbf{K}]$ - the proportional damping matrix with coefficients a_1, a_2; $[\mathbf{C}_T]$ - heat capacity matrix; $[\mathbf{K}_T]$ - heat conductivity matrix; $[\mathbf{H}]$ - thermal-elasticity matrix.

After eigenvalue problem (1) is solved, the obtained complex eigenvalues define the Q-factor of the structure as $Q = \left|\frac{\mathrm{Im}(\lambda)}{2 * \mathrm{Re}(\lambda)}\right|$.

The numerical analysis techniques developed along with experimental methods allow for a comprehensive characterization of laminated MEMS structures. During the project LTU-AVT-07/1 a new FE model was developed in order to evaluate the thermal-elastic damping in SiO_2-Si-Cu cantilever beam. The distribution of amplitude values of temperature at 1st in-plane and 3rd out-of plane bending modes of the beam vibration are depicted respectively in Fig 1 and Fig. 2.

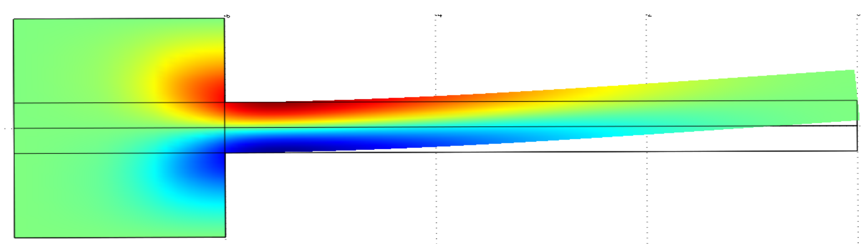

Figure 1. The distribution of amplitude values of temperature at 1st in-plane mode of the SiO_2-Si-Cu cantilever beam.

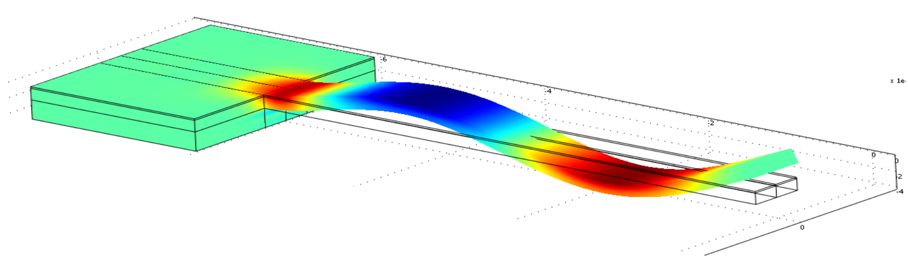

Figure 2. The distribution of amplitude values of temperature at 3rd out-of plane bending mode of the SiO_2-Si-Cu cantilever beam.

The dependencies of modal frequencies and quality factors of the MEM resonator on the lamina thickness are shown in Fig. 3 and Fig. 4.

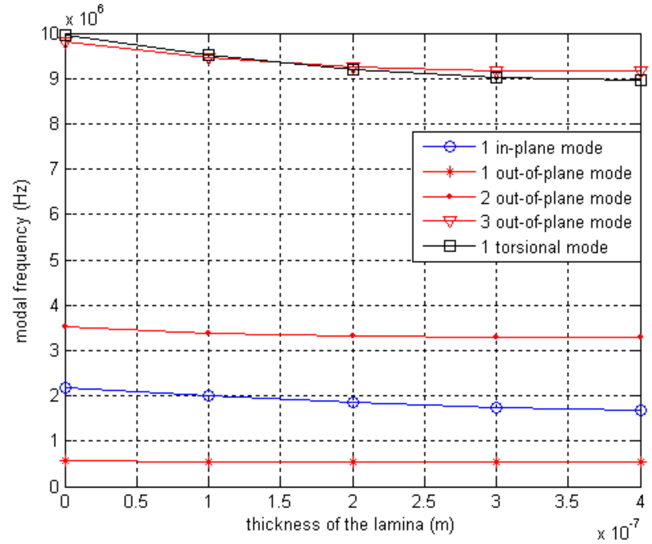

Figure 3. Modal frequencies of the MEMS resonator vs. the lamina thickness.

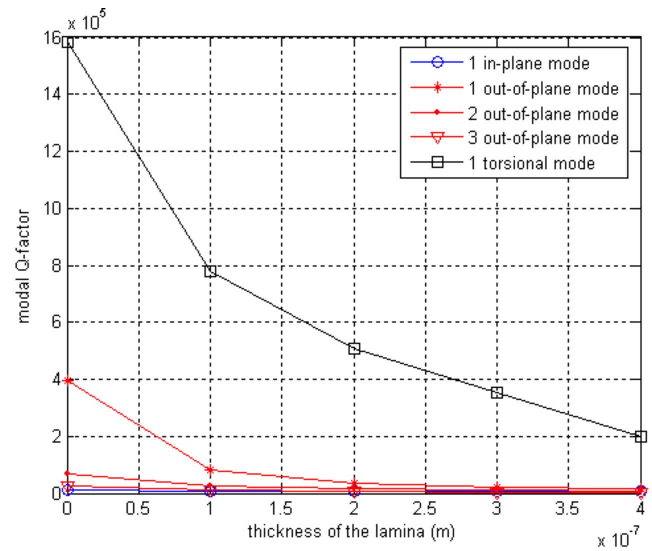

Figure 4. Modal Q-factors of the MEMS resonator vs. the lamina thickness.

3. MODEL VERIFICATION

A significant part of any simulation is the verification and validation of the model. It is necessary to be sure that the model design (conceptual model) is transformed into a computer model with sufficient level of adequacy and accuracy.
In this study, a sample model of a cantilever beam resonator was investigated numerically and experimentally. Three modifications of the sample cantilever resonator numerical model were explored:

a. close-to-reality clamping conditions and the presence of the SiO_2 substrate were taken into account. Full clamping and prescribed temperature (ambient temperature T_0) boundary conditions were imposed on all cut-boundaries that separate the computational model form the remaining body of the MEMS. The model included the resonating structure and a certain part of the surrounding domain. The mechanically free surfaces of the model were assumed to be thermally isolated;

b. the cantilever resonator was presented without the surrounding material. One end of the resonator was fully clamped, and fixed temperature boundary conditions were imposed at this end;

c. the same model as in (b), only that the clamped end was thermally isolated.

Table 1 presents the first modal frequency and Q- factor values of the three models of the SOI cantilever resonator at three different values of its length. Model modification (a) is close to the situations observed in a real MEMS. However, analytical formulae are able to evaluate the situations b and c only, therefore they are used for comparison of the results and model verification.

The thermal boundary conditions influence the TED effects (i.e., the Q-factor values). Ideally clamped resonator models as in modifications (b) and (c) give higher natural frequencies values compared with ones provided by investigation of more realistic model (a).

Table 1. Modal frequency and Q-factor values of the sample cantilever resonator

Length (m)	Model a f (MHz), Q	Model b f (MHz), Q	Model c f (MHz), Q
0.5×10^{-5}	48.12MHz; 1.347×10^{4}	54.3MHz; 1.472×10^{4}	54.3MHz; 1.074×10^{4}
1×10^{-5}	13MHz; 4.13×10^{4}	13.8MHz; 4.6×10^{4}	13.8MHz; 4.6×10^{4}
2×10^{-5}	3.358MHz; 1.612×10^{5}	3.46MHz; 1.736×10^{5}	3.46MHz; 1.549×10^{5}

Fig 5 demonstrates the comparisson of the first modal frequency and Q-factor values obtained by using FE models against analytical evaluations. The model parameters were matched to the design data of poly-Si and poly-C resonators given in[11].

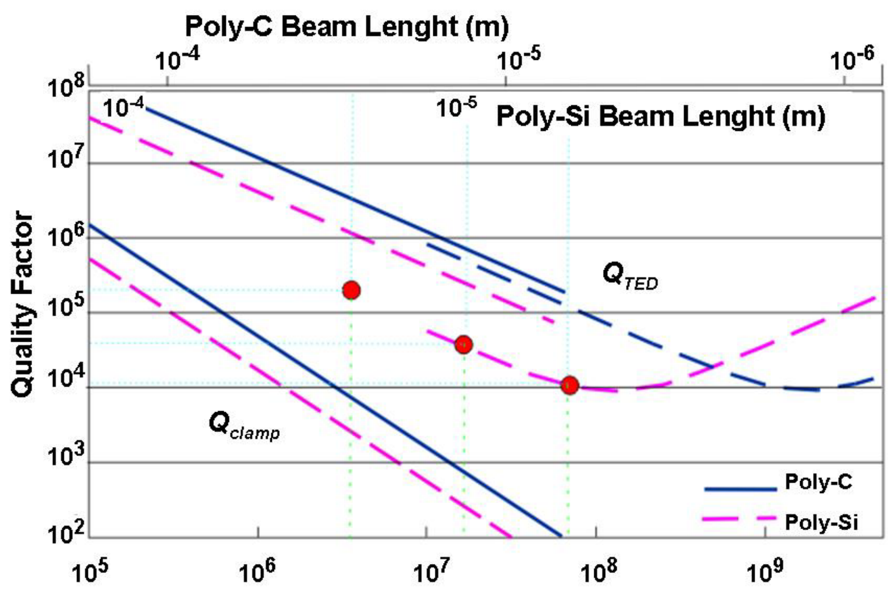

Figure 5 Dependencies of Q-factor values determined by different vibration energy loss mechanisms against the resonant frequencies of poly-Si and poly-C cantilever resonators[11]; red dots indicate the numerical results obtained in this work by FE analysis for poly-Si cantilever resonatoprs of length $9\times10^{-4}m,\ 2\times10^{-5}m,\ 6.5\times10^{-5}m$.

The resonators[11], were fabricated and tested using piezoelectric and electrostatic actuation methods in low and high vacuum. Fig. 5 presents the dependencies of the Q-factor values determined by different vibration energy loss mechanisms against the resonant frequencies of poly-Si and poly-C 1µm thick cantilever beam resonators.
The simulation results were also validated experimentally in[7] by testing a cantilever beam resonator. The dependence of the numerically obtained values of the Q-factor of 1st damped mode against temperature is depicted in Fig. 6.

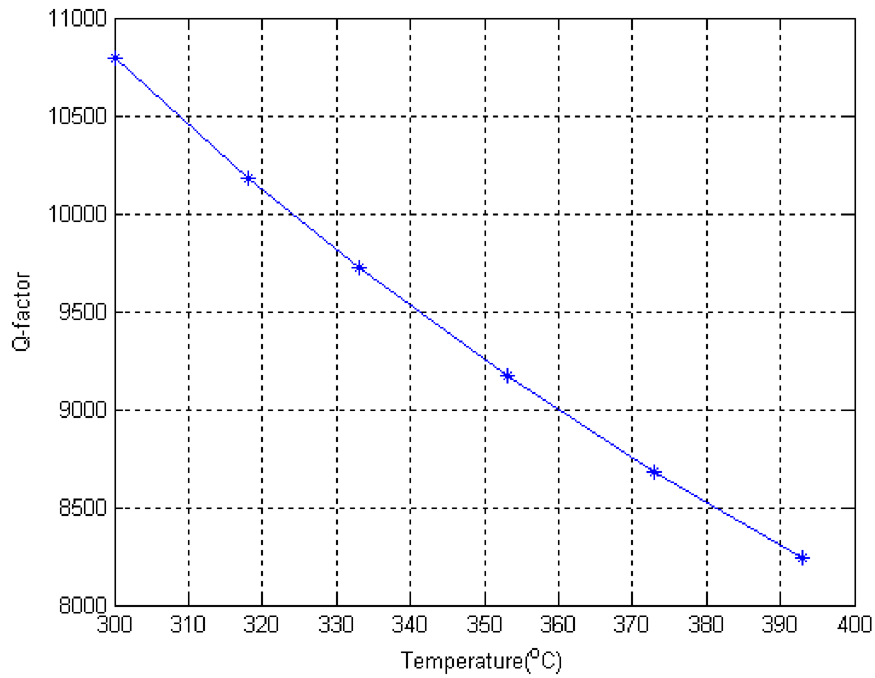

Figure 6. The dependence of numerically obtained values of the Q.

It can be admitted that the qualitative agreement of numerical and experimental results is good (about 2500 drop of the Q-factor value over temperature range of 100 °C in both cases). However, the experimentally obtained Q-factor value is about 2700 higher over all temperature range. It seems reasonable to conclude that the difference of values is caused by other damping mechanisms not represented by purely TED model. Simultaneously, the obtained values proove the major signifficance of TED during vibrations of MEMS resonators, as the value of the Q-factor caused by TED is approximately 8000-10500 against 5500-8000, which is obtained if simultaneously other energy dissipation mechanisms are taken into account.

4. CONCLUSIONS

The Q-factor determined by thermal-elastic damping of micro-electro-mechanical resonators structures is a very important dynamic characteristic since it provides the upper limit of the Q-factor that is possible to achieve in a structure of given geometry and materials under an assumption that no internal friction and other sources of damping are present. A FEM computational model of longitudinal and bending vibrations of a beam resonator has been developed in order to analyze the eigenfrequencies and Q factors of test vehicle as well as real MEMS structures. Model verification has been performed by calculating modal properties of unsupported beam structures and comparing against the analytically obtained results recently published by other authors.
The comparison of calculated and experimentally obtained resonant frequencies and Q-factor values indicated a good agreement of tendencies of change of the quantities against temperature. Both the experiments and calculations revealed almost linear decrease of the Q-factor against temperature. However, all experimental Q- factor values were lower than

theoretical ones. The shift of the values could be explained by other mechanisms of damping, which are not included into the thermal-elastic damping model. Their influence can be quantitatively evaluated by the said difference of the values.

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